

# "Dost thou love Life? Then do not squander Time, for that's the Stuff Life is made of." 

\author{

- Ben Franklin
}


## -Lume ■S\&P 500



Worth of Initial (2010) Dollar Invested at Year End (\$)


All figures pretax. Excludes fees of hypothetical S\&P 500 index fund investment.

Lume Group's net liquidation value grew $34.1 \%$ in 2019, compared with a total return of $31.5 \%$ for the S\&P 500. Over nine years, Lume Group's CAGR was $16.8 \%$, meaning that a dollar invested at the end of 2010 would be worth a little over four dollars at year end 2019 on a pretax basis.

## On Misquotes: A Correction

I began this series of annual reports with a mistake in the 2015 letter. I misattributed a quote to Ben Franklin, so a statement that forms the foundation of Lume was ironically made in error. It was never actually said by Franklin despite my attribution of his name to it-there are no reliable sources linking Franklin to the line:
"An investment in knowledge pays the best interest."

## - Not Ben Franklin

This misattribution reminds me of another quote I like that Einstein also never (but is purported by many sources to have) said:
"Compound interest is the eighth wonder of the world. He who understands it, earns it; he who doesn't, pays it."

- Not Albert Einstein

These are quotes that were posthumously attributed to great thinkers, perhaps to add to the power of their message. I don't believe that either requires any such boost, for the remarks crafted by unknown individuals are inherently profound.

I'll acknowledge my mistakes whenever I note them, but won't dwell on them. I do try to learn from my mistakes, so more rigor on my part in checking the validity of information derived from online sources is warranted (the internet is full of falsehoods these days and significantly more caution is required for any truth-seeker). In the spirit of the actual Ben Franklin quote in this report, I've taken note and will now move on.

## Ergodicity \& One Hundred Cousins

I said in last year's letter that if a roulette wheel with only black and red slots (50:50 odds) pays you 2 chips if you choose correctly (200\% return) and costs only 1 chip if you are wrong ( $100 \%$ loss-an asymmetric payoff structure), you should take that bet every single time. But then there's another question to ask here-how much should you bet? What proportion of your net worth? Should you put it all on the line?

Most economists will say the rational choice is to take on the bet, and that anyone who refuses the bet is irrational, perhaps that such an individual is misguided due to "loss aversion". However, few mention how much one should bet. Imagine if you bet your entire net worth on a single spin and lost-you'd be broke ("ruined") and unable to take part in future spins of the wheel. So, if you were told by this casino that you must bet your entire net worth and nothing less, the rational thing to do is to turn down the bet. You are harmed more by the loss than you'd benefit from the gain (a steep asymmetry in outcomes).

But what if with each spin you could choose how much to put down? What is the optimal bet size that will deliver the most returns over time while (and this is of the utmost importance) avoiding ruin for the individual)? There is a number between $0 \%$ and $100 \%$ of your net worth that you ought to bet on each spin. The Kelly Criterion helps solve for that number.

Before we get into Kelly, let's consider how economists typically look at the roulette situation. Usually, they consider the average return of a large number of individuals - "an ensemble". Unfortunately, what is often missed when one looks at the average in a population is that there are certain individuals who hit ruin while a small minority achieve vast prosperity. By nature of the simple mathematical computation known as the average, the spectacular successes of a few can drive the average higher even if most people in the group are ruined.

Imagine if you took a random group of twenty individuals from Scranton, PA, put them in a room, and found the average net worth of those individuals. Now imagine if Jeff Bezos walked into the room. What would happen to the average? Would the resulting number tell you anything useful about the 20 Scranton residents in the room? As a matter of fact, it would become a useless measure of how those 20 individuals were doing. In a similar fashion, extreme levels of income inequality can also distort figures such as GDP per capita. The median, not the average, would be a much more useful measure in this scenario.

So when economists look at what happens to a group - or "ensemble" - they look at the average outcome, potentially missing something important: that sometimes even most of the individuals in a group where the average is high are ruined. These are what are termed non ergodic situations.

Ole Peters, a professor at the London Mathematical Laboratory, has a blog exploring this phenomenon called Ergodic Economics. Peters and Murray Gell-Mann illustrated the important nuances between observing an ensemble versus an individual subject to repeated bets over the course of time. This is a concept I first formally encountered in Nassim Nicholas Taleb's Skin in the Game (the final chapter), but it is one Taleb has covered in other ways, often with examples or heuristics throughout
his works (one example is the big Russian Roulette contest in "Fooled by Randomness").

I will do my best to expand on the concept here as it pertains to investing, but note that there is a YouTube video by Peters called "TEDxGoodenoughCollege - Ole Peters - Time and Chance" that is worth watching to understand this concept.
"In order to succeed, you must first survive"

- Warren Buffett

In our roulette wheel that pays 2 chips and costs 1 chip with 50:50 odds of either outcome, if we were to bet $100 \%$ of our net worth with each spin, we are guaranteed to hit ruin (our uncle point) within just a few spins. Remember that the expected value (EV) of this bet is positive:

$$
E V=0.5 *(2)-0.5 *(1)=0.5
$$

Despite the expected value of the situation being positive, betting $100 \%$ of our net worth on each spin would inevitably lead to a spin where we lose our bet (entire net worth) with $50 \%$ probability. So, it is unwise to put all our money on the line for such a bet.

A similar situation exists for a less obvious case. Imagine a roulette wheel pays 0.5 of your bet and costs 0.4 of your bet based on what you bet. Again, EV is positive:

$$
E V=0.5 *(0.5)-0.5 *(0.4)=0.05
$$

However in this situation, what happens if we wager our entire net worth which starts at $\$ 100$ on each spin over 100 spins? Let's observe a single individual who does so over time:

Figure 1: Sample Trajectory


It becomes apparent that over time, you lose everything ("ruin"). The first spin was profitable and you were up $50 \%$ of $\$ 100$ or $\$ 150$, but on the second spin, you bet the $\$ 150$ you had accumulated and lost, so you lose $40 \%$ of $\$ 150$ or $\$ 60$ leaving a remaining net worth of $\$ 90$, or less than what you started with after 1 win and 1 loss. This randomly continues onward until net worth dwindles to effectively zero.

You can repeat this experiment in any of your favorite computational software tools (or in a spreadsheet) and see that "ruin" is
inevitably true for most individuals. However, there is a minority of individuals that sees spectacular success and their wealth "blows up":

Figure 2: Success Trajectory


This individual's wealth, starting at the same humble $\$ 100$ and subject to the same odds and payoffs of the roulette wheel as before, spikes as high as $\$ 987,618.76$ before ending up at $\$ 76,797.24$ after the 100th spin. This individual is a rarity, but what is important to note is that if we imagined 20 individuals engaged in spins of the roulette wheel and looked at the "ensemble" or average, this individual's results, like Jeff Bezos' net worth, would cause the average ensemble outcome to be equal to $\$ 3,839.86$ at the end of 100 spins even if all 19 of the other participants had gone to ruin (the median however would be $\$ 0$ ). If you were observing the ensemble, it would look like wealth of the 100
individuals were growing over time, but you would miss that 19 individuals went bust and there was only 1 wild winner.

Now what if we observed the individual net worths of more people - 100 of our cousins, engaged in this game?

Fig 3: 100 Cousins Playing a Game


Here you see at the end two cousins who are doing well and most who have reached ruin near zero net worth. So, only 2 cousins have seen success with 100 positive EV bets over time and the average of everyone "the ensemble" looks good, but the average is misleading.

So let's look at this like social scientists/economists allegedly do with their simplification and take the average of the 100 cousins:

Fig 4: Ensemble of 100 Cousins Playing a Game


It looks like the average is steadily growing-something we'd expect from a positive EV scenario, but the devil is in the details-it looks like everybody's wealth is increasing, but we know under the hood that it is likely just a few individuals driving the average higher while the vast majority of our cousins have gone bust ("ruined").

This type of situation-where the average does not give you the true picture of what happens to individuals over time-is deemed non ergodic.

What happens to this growing average (ensemble) net worth if we continue with our 100 cousins on out much further-say with 1,000 spins?

Fig 5: Ensemble of 100 Cousins Playing a Game for $\mathbf{1 0 0 0}$ Spins


Now we see that if we keep the game going on longer and longer, eventually everyone hits zero. All are converged at ruin. There are a few spurts of positive surges, but eventually the average goes to a solid zero (unlike at other values, in the case when the average equals zero, it tells us everything we need to know about all the individuals in the ensemble). So we see the end result of betting it all, even when the odds and/or payoffs favor us. So, if we should not put all our money on the line, how much should we bet?

## Position Sizing \& The Kelly Criterion

> "The whole secret of investment is to find places where it's safe and wise to non-diversify. It's just that simple. Diversification is for the know-nothing investor; it's not for the professional."

- Charlie Munger

How can we actually apply the lessons of these simulations to real life? Should we avoid wagering large sums in individual situations? Meanwhile, Charlie says wide diversification is not for the professional investor, but our simulations show that if you concentrate all of your chips on a situation even with favorable odds and payoffs, you are likely to hit your uncle point (ruin).

In investing, it is impossible to calculate odds, and what's more is that odds of an investment's outcomes are forever changing with time as the world provides more information about the prospect and as external factors such as interest rates change (so the odds should be frequently updated using Bayesian Inference as discussed in our 2017 letter).

The conflict between our experiments and the advice against diversification is a mirage. What Munger and Buffett are telling us is 1) invest where you are confident in the odds being in your favor (i.e. inside your "circle of competence"), 2) invest where the payoffs are in your favor (i.e. with an adequate "margin of safety"), 3) do not widely diversify (more on this later) if you are a professional capable of \#1 and \#2, and 4) do not leverage-or borrow-to invest (which only accelerates ruin over time), but rather have some cash on hand (the anti-fragile asset class). This will ensure ruin avoidance - or survival - which is a prerequisite for success.

So why not diversify to mitigate the risk of ruin? We should certainly diversify up to a certain point, but evidence shows that beyond the
number of investments that you can count on your hand, the benefits of diversification diminish. What's more is that it matters what you are concentrated in. Are you concentrating in a business or two with robust earnings (possessing a moat, non-cyclical), large net cash position and management capable of optimal capital allocation, or one with a large net debt position and cash flows highly subject to the whims of the business cycle or competitive price wars? It's not just about how you allocate capital in your portfolio, but how your businesses earn and allocate their own capital. So for the professional who is capable of assessing these things, wide diversification does not make sense.

The Kelly Criterion tells us how much we should bet. It tells us that if we have increasingly favorable odds and/or payoffs in a situation, we should bet more on that situation. And Kelly tells us that this only works up to a certain point and $100 \%$ allocation will generally lead to ruin. And here is a derivation of this analysis which I credit to Nassim Nicholas Taleb who has made this into a digestible and presentable form for me.

Assume our roulette wheel now has symmetric payoff (it pays 1 chip if you are correct and costs 1 chip if you lose). So, each time we win, we accumulate $100 \%$ growth in our investment and each time we lose, 100\% loss. Now, let's vary our probability of winning the spin " $p$ " and see what the optimal allocation is based on our probability of winning (so we are fixing our payoff at symmetric level and changing our odds). The following math section is for math geeks only, for all others-skip the next page and move directly to Figure 6.

## Math: Kelly Analysis Derivation

$A=P(1+r)^{N}$, the compound growth formula, is rearranged to:

$$
\text { (i) } 1+r=(A / P)^{1 / N} \text {, }
$$

where $r$ is growth rate, $A$ is amount after compounding, $P$ is principal or starting capital, and $N$ is number of periods.

With our roulette wheel, we also have,

$$
\text { (ii) } A=P(1-l)^{L}(1+l)^{W} \text {, }
$$

where $W$ is number of wins, $L$ is number of losses, and $l$ is the fraction of principle won or lost with each bet (percent of principal allocated).

Substituting (ii) into (i) gives: (iii) $1+r=\left((1-l)^{L}(1+l)^{W}\right)^{1 / N}$

Note that over the long haul (law of large numbers), $W / N$ converges to probability of winning $p$ while $L / N$ converges to probability of losing $1-p$, so (iii) becomes:
(iv) $1+r=(1-l)^{1-p}(1+l)^{p}$ and taking the log of both sides of (iv):
(v) $\log (1+r)=(1-p) \log (1-l)+p \log (1+l)$

And finally, taking the exponent of both sides and rearranging gives $\uparrow$ :

$$
\uparrow r=\exp ((1-p) \log (1-l)+p \log (1+l))-1
$$

Now, we take equation $\downarrow$ and plot $r$ as a function of wager size (or percent allocation) $l$ for different win probabilities $p$ (Figure 6):

Fig 6: Kelly Analysis For Different Winning Probabilities p


As you might expect, with increasing probability of winning (a larger "edge"), one ought to allocate more of a portfolio to bets. What is not so clear is that, with the exception of a sure thing ( $100 \%$ win probability), the long term growth rate goes negative, and to zero (ruin) if you over-allocate. So, any time there is uncertainty, even if the EV of the bet is positive, overallocation leads to ruin. This is the same result as our simulations before except instead of favorable payoffs, we have favorable odds (either way, it leads to positive EV).

The optimal bet size occurs for each of these curves (except $p=1$ ) at a maximum which is calculable by setting the derivative with respect to $l$ (percent allocation) to zero which we will leave to the engaged reader to complete (the solution is the optimal bet size given by the Kelly Formula: $l=2 p-1$ ). So, even at $75 \%$ probability of winning (which you may go a lifetime without encountering in the real world of investing), the optimal allocation is $50 \%(l=0.5)$.

This roulette example is not necessarily generalizable to every investment situation. Obviously exact odds (probabilities) of success (winning) are unknowable and the difficulty of estimating them compounded by the need to frequently update the odds as the world produces new information (Bayesian Inference). Remember also that most stock purchase investments are by their nature asymmetric-the downside is limited to $100 \%$ while the potential upside is beyond this number which makes the game more favorable from a payoff standpoint than our clear cut roulette example. This makes it difficult to map Kelly precisely onto common stock investing.

Diversification can help you mimic more of the ensemble rather than the individual. By buying multiple securities, say 4 companies with $62.5 \%$ edge (and symmetric +/- 100\% payoffs), you allocate $25 \%$ to each one, you can represent 4 investors instead of 1 and now what happens to the ensemble may be very relevant to what happens to you the individual.

In such a scenario, one must consider the independence of risks when diversifying among a handful of favorable investments. For example, if you divide your net worth equally into four automakers (or four oil producers) and an economic downturn hits, you will receive a firsthand lesson on the pitfalls of correlation and importance of ensuring independence of risk among investments (just as an insurance company that only insures houses on a single island in the Bahamas will learn at
some point). Diversification does not help you if your investments are strongly correlated and you will one day feel the same pain as someone who put it all into one automaker (or oil company).

Charlie Munger does not widely diversify because he chooses a few bets in his circle of competence he is highly confident about and (consistent with Kelly), bets aggressively on them, but not too aggressively. And while there is high concentration in a single security among many large Berkshire Hathaway shareholders, that company itself is highly diversified conglomerate, always with a net positive cash position (so it has diversification and anti-fragility embedded inside of it). Another key to success with concentration is if the facts change, you change your mind: update the model rigorously (e.g. exit the newspaper business if there is no future). Again, it's for the professional investor.

So, as a debate as to the merits of behavioral economic theory of loss aversion versus those who understand ergodicity rages, we will merely practice the simple rule of allocating more to positions we are more confident in, but never over-allocating. And we will always remember the profound wisdom ingrained in the line:
"In order to succeed, you must first survive."

- Warren Buffett


## Conclusion: A North Star in a Volatile World

> "In my whole life, I have known no wise people who didn't read all the time - none, zero."

- Charlie Munger

Few businesses or endeavors can survive the punishing volatility that inevitably comes with the test of time. I aim to learn from those who managed this very achievement. Whether it's large scale conflicts, pandemics, or panics, my aim is simple - to survive and thrive. The knowledge I pursue-in reading, truth-seeking-helps unlock key insights on how to accomplish this feat. Wisdom is the hopeful byproduct from what will be a lifelong process.

Signed,
P. Dalal

